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THEORETICAL DESCRIPTION OF THE PHENOMENON OF
LOSS OF FLUIDITY IN POLYMER LIQUIDS SUBJECTED TO
INTENSIVE DEFORMATION

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Polymer liquids display a number of properties characteristic of solids: slippage along the wall, the appearance of cracks in the material during flow, brittle fracture under tension, etc. The combination of these phenomena encountered in the flow of polymers through capillaries, accompanied by a number of other effects (oscillations and waves on the surface of a jet emerging from a capillary, crystallization of polymers in a capillary, etc.), has been referred to in the literature as "destruction of melt." The bibliography devoted to this question, which is important for many polymer-processing methods, is very extensive (see, for example, [1]). The behavior of polymer liquids has been observed recently for melts of polymers with a narrow molecular-weight distribution (MWD) [2, 3] for conventional types of deformation. In [4] the hardening effect was studied for the case of the extraction of polyoxyethylene from a tank by means of a rotating drum. The length of the liquid jets so obtained was as much as half a meter. In the present study we propose a theoretical description of the above-mentioned effects for two of the situations most often found in practice: simple shear and simple tension.

§1. A theoretical description of the phenomenon of loss of fluidity in polymer liquids and their transition to a highly elastic state will be considered in the simplest three-constant nonlinear model of an elastoplastic medium of Maxwellian type, proposed in [5],

$$\sigma = -p\delta + 2CW_1 = 2C^{-1}W_2 \quad (W_j = \partial W / \partial I_j); \quad (1.1)$$

$$C^{\nabla} - C\epsilon - \epsilon C + 2C\epsilon_p(C) = 0, \quad \text{spe} = 0, \quad \det C = 1; \quad (1.2)$$

$$e_p = (2\lambda_*(T)) \exp\{-\beta \int_{t_0} W_*\} [(C - \delta T_1/3)W_{*,1} - (C^{-1} - \delta I_2/3)W_{*,2}]; \quad (1.3)$$

$$I_1 = \text{sp}C, \quad I_2 = \text{sp}C^{-1}, \quad W = \rho_0 f(T, I_1, I_2), \quad 2W_* = W(I_1, I_2) \div W(I_2, I_1); \quad (1.4)$$

$$D = \frac{1}{3\lambda_*(T)} \exp\left\{-\frac{\beta}{\rho_0} W_*\right\} \{(I_1 I_2 - 9)(W_1 W_{*,2} + W_2 W_{*,1}) + 2(I_1^2 - 3I_2)W_1 W_{*,1} + 2(I_2^2 - 3I_1)W_2 W_{*,2}\}, \quad (1.5)$$

where $C^{\nabla} = (\partial/\partial t + v_{\alpha}\partial/\partial x_{\alpha})C + \omega C - C\omega$.

Here we give the rheological equations of the model (1.1)-(1.3) for an incompressible liquid in a Cartesian coordinate system; σ is the stress tensor; p is the isotropic pressure; ϵ is the tensor of deformation rates; ω is the vortex tensor; the symmetric positive-definite tensor C represents the elastic deformation (Finger measure) accumulated during the motion of the elastic liquid; e_p is the tensor of the irreversible rate of deformation; δ is the unit tensor; I_1, I_2 are independent variants of the tensor C ; f is the specific free energy; W is the elastic potential; D is the dissipation function; C^{∇} is the Jaumann derivative of the tensor C with respect to time; and T is the temperature.

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The connection between the stress tensor σ and the elastic deformation tensor \mathbf{C} , in accordance with formula (1.1), is of the same form in this model as in the case of a nondissipative incompressible isotropic elastic medium. From considerations of thermodynamic stability, $W_j \geq 0$.

The first (tensor) equation (1.2) corresponds to the kinematic relation between the reversible and irreversible rates of the deformations. The other two scalar relations in (1.2) are the incompressibility conditions.

The expression for the tensor of irreversible deformation rates e_p , defined by formula (1.3), in accordance with the concept of a nonlinear Maxwellian liquid, depends only on the reversible deformation, the tensor \mathbf{C} . This expression has the following properties [5]:

$$e_p = -q\delta + 2C\partial\psi/\partial C, \quad \psi = (\mu_0/\beta\lambda_*)[1 - \exp(-(\beta/\mu_0)W_s)]; \quad (1.6)$$

$$\|e_p\| \rightarrow 0, \quad \|\mathbf{C}\| \rightarrow \infty \quad (\|A\|^2 = spA^2). \quad (1.7)$$

The relation (1.6) means that there exists a nonequilibrium potential ψ , dependent only on the elastic potential W_s (1.4), which is symmetric with respect to the arguments I_1, I_2 ; the scalar quantity q is a Lagrangian multiplier found from the incompressibility condition $spe_p = 0$ (1.3).

The property (1.7) of formula (1.3) corresponds to the assumption, advanced in [5], that for sufficiently large reversible deformations there is a loss of fluidity in the system owing to the intensive growth of the characteristic scalar relaxation time of the system as a result of the orientation of the macromolecules. As can be seen from (1.5), $D \rightarrow 0$ as $\|\mathbf{C}\| \rightarrow \infty$, i.e., all the rheological relations are transformed into relations for a nonlinearly elastic medium.

Lastly, an important property of formula (1.3) for e_p is the anisotropic relation between σ and e_p for finite reversible deformations \mathbf{C} (so-called forced anisotropy), which describes the processes of orientation taking place during intensive flow of polymer liquids. Forced anisotropy (of this type) manifests itself not only in the viscous and relaxation properties of flowing polymer systems, but also in the phenomena of heat conduction, diffusion, polarizability, etc.

In the present study we shall make use of the simplest elastic potential of the classical statistical theory of high elasticity [6]:

$$W = \mu_0(T)(I_1 - 3), \quad \mu_0 \sim \rho_0 RT/\mu_c, \quad (1.8)$$

where R is the gas constant; μ_c is the average molecular weight of the chain segment between two joints. In this case

$$W_1 = \mu_0; \quad W_2 = 0; \quad W_s = (\mu_0/2)(I_1 + I_2 - 6); \quad W_{s,1} = W_{s,2} = \mu_0/2. \quad (1.9)$$

Thus, the model considered above describes the nonlinear viscoelastic effects caused by the existence of enormous reversible deformations in flowing polymer systems. In the model, qualitative account is taken of considerations involving the structure of flowing polymers as a fluctuating-net structure; the loss of fluidity is treated as a relaxation transition of the polymer system into a highly elastic state.

The relations (1.1)-(1.3) (in the isothermal formulation considered below) contain three material constants: $\lambda_*(T)$, $\mu_0(T)$, β . The parameters λ_* and μ_0 of the liquid are expressed linearly in terms of the maximum Newtonian viscosity η_0 and the relaxation time θ_0 :

$$\lambda_*(T) = 2\eta_0(T); \quad 2\mu_0 = \eta_0/\theta_0. \quad (1.10)$$

The numerical parameter β ($0 < \beta < 1$) characterizes the flexibility of the macromolecules, increasing as their rigidity increases, and determines the essentially nonlinear viscoelastic properties of the material.

A system of equations more complicated than the system (1.1)-(1.3) was used in [7, 8] for describing the nonlinear properties of flowing polymers under simple shear for reversible deformations which are not very large ($\beta \ll 1$) far from the transition to the highly elastic state; the results were found to be in good agreement with stationary and nonstationary shear experiments both for tangential and for normal stress. We shall consider below the behavior of a polymer liquid for finite values of β , concentrating our attention on the transition from the fluid to the highly elastic state.

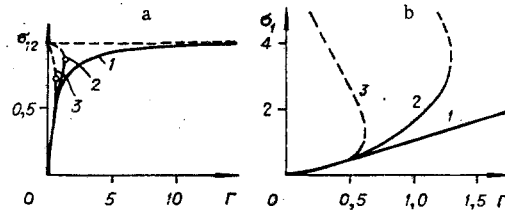


Fig. 1

§2. We consider the case of simple shear. The kinematic matrices have the form

$$\begin{aligned}
 \mathbf{e} &= \dot{\gamma}/2 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; & \boldsymbol{\omega} &= \dot{\gamma}/2 \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\
 \mathbf{C} &= \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}; & \mathbf{C}^{-1} &= \begin{pmatrix} C_{22} & -C_{12} & 0 \\ -C_{12} & C_{11} & 0 \\ 0 & 0 & 1 \end{pmatrix},
 \end{aligned} \tag{2.1}$$

where $\gamma = \gamma(t)$ is the rate of deformation (these were determined on the basis of adhesion conditions).

From (2.1) we have $I_1 = I_2 = 1 + C_{11} + C_{22}$. The incompressibility condition for irreversible deformations $\det \mathbf{C} = 1$, on the basis of (2.1), can be written in the form

$$C_{11}C_{22} = 1 + C_{12}^2.$$

Making use of (1.8)-(1.10) and substituting (2.1) and (1.3) into the tensor relation (1.2), we obtain $[\tau = t/\theta_0, \Gamma(\tau) - \theta_0 \dot{\gamma}(t)]$

$$\begin{aligned}
 2 \frac{dC_{11}}{d\tau} + (C_{11}^2 + C_{12}^2 - 1) e^{-\beta w} &= 4\Gamma C_{12}; \\
 2C_{11} \frac{dC_{12}}{d\tau} + C_{12}(C_{11}^2 + C_{12}^2 + 1) e^{-\beta w} &= 2\Gamma(1 + C_{12}^2); \\
 C_{22} = C_{11}^{-1}(1 + C_{12}^2), & \quad w = W/\mu = C_{11} + C_{22} - 2.
 \end{aligned} \tag{2.2}$$

The dimensionless tangential stress and the two normal-stress differences have the form

$$\sigma'_1 \equiv \frac{\sigma_{11} - \sigma_{22}}{2\mu_0} = C_{11} - C_{22}; \quad -\sigma'_2 \equiv \frac{\sigma_{33} - \sigma_{22}}{2\mu_0} = 1 - C_{22}; \quad \sigma'_{12} \equiv \frac{\sigma_{12}}{2\mu} = C_{12}.$$

The dimensionless dissipative function is given by the expression

$$D' = \frac{2D\theta_0}{\mu_0} = \frac{(C_{11}^2 - C_{12}^2 - 1)^2 - 4C_{11}^2}{2C_{11}^2} e^{-\beta w}.$$

From this point on we shall omit the primes after the variables. We consider two fundamental problems. The first is the problem of the establishment of a stationary flow regime for $\Gamma = \text{const}$ from a state of rest with the initial conditions

$$C_{11}|_{t=0} = 1; \quad C_{12}|_{t=0} = 0. \tag{2.3}$$

The second is the problem of relaxation of the stresses ($\Gamma = 0$ when $t > t_0$)

$$C_{11}|_{t=t_0} = C_{11,0}; \quad C_{12}|_{t=t_0} = C_{12,0} \quad (C_{11,0} > 0, \quad C_{12,0} > 0).$$

Before considering the nonstationary solutions of the system (2.2), we shall consider its stationary solution, which can be conveniently represented in the form

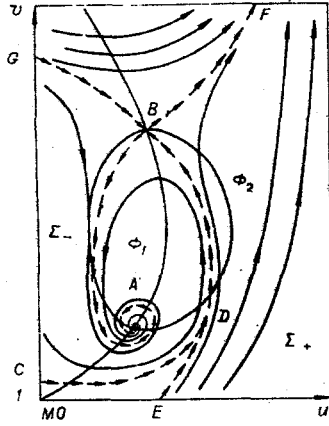


Fig. 2

$$\begin{aligned}
 C_{11}^0 &= \frac{x^2 + 1}{\sqrt{1 - x^2}}; & C_{12}^0 &= x; & C_{22}^0 &= \sqrt{1 - x^2}; \\
 \Gamma^0 &= \frac{x}{1 - x^2} \exp \left[2\beta \left(1 - \frac{1}{\sqrt{1 - x^2}} \right) \right]; \\
 \sigma_1^0 &= \frac{2x^2}{\sqrt{1 - x^2}}; & -\sigma_2^0 &= \frac{x^2}{1 + \sqrt{1 - x^2}}; & \sigma_{12}^0 &= x.
 \end{aligned}
 \tag{2.4}$$

From (2.4) it follows that the parameter $x \equiv C_{12}^0$ varies in the interval $[0, 1]$ and the functions giving the stationary tangential and normal stresses in terms of Γ^0 will be two-valued and will exist when $0 < \Gamma^0 \leq \Gamma_m$, where $\Gamma_m(\beta) = \max \Gamma^0(x, \beta)$ from (2.4).

Figures 1a and 1b show, respectively, the curves of $\sigma_{12}^0(\Gamma_0)$, $\sigma_1^0(\Gamma_0)$, calculated for a number of values of β [1] 0.1; 2) 0.5; 3) 1.0] according to the formulas in (2.4).

Let us consider the question of the stability of the stationary solution (2.4) of the system of equations (2.2) with respect to small perturbations. We set

$$C_{11} = C_{11}^0 + ye^{xt}; \quad C_{12} = C_{12}^0 + ze^{xt}.$$

Substituting these equations into the equations (2.2), which are linear for small perturbations, we obtain a system of linear homogeneous equations for y and z . Setting the determinant of this system equal to zero, we obtain the characteristic equation for the rate of growth of the perturbations χ , the solution of which can be represented in the form

$$e^{\beta\omega\chi_{1,2}} = -(1/\sqrt{1-x^2}) + [\beta x^2/(1-x^2)] \pm (x/\sqrt{1-x^2}) \sqrt{x^2(1+\beta^2)-1}, \tag{2.5}$$

where x is found from (2.4) ($0 < x < 1$) and is a dimensionless tangential stress.

It follows from (2.5) that for $0 < x < (1 + \beta^2)^{-1/2}$ the singular point of the system (2.2) is a stable focus; for $(1 + \beta^2)^{-1/2} < x < x_*(\beta)$ it is a stable node; for $x_*(\beta) < x < 1$ it is a saddle. Here the quantity $x_*(\beta)$ is the positive root of the equation

$$(1 - x_*^4)(1 + x_*^2) = 4\beta^2 x_*^4. \tag{2.6}$$

We can convince ourselves that to the value of x_* from (2.6) there corresponds a maximum of $\Gamma^0(x, \beta) = \Gamma_m(\beta)$ from (2.4). Thus, we have shown that the two-value functions $C_{12}^0(\Gamma)$, $C_{11}^0(\Gamma)$, have a region of stability corresponding to the lower branches of these curves; the upper branches of the curves are unstable. As the constant parameter Γ increases for a fixed β , the nonstationary curves $\sigma_{12}(\tau)$, $\sigma_1(\tau)$ will assume a steady state with oscillations which disappear in the region near $\Gamma_m(\beta)$. For $\Gamma > \Gamma_m$ there is no stationary regime of flow. In this region of the values of the parameter Γ , with the adhesion conditions preserved, the stresses increase beyond all bounds as the dimensionless-time variable τ increases, and for large values the following asymptotic equations hold:

$$C_{12} \approx \Gamma\tau; \quad C_{11} \approx 1 + \Gamma^2\tau^2 \quad (\tau \gg 1), \tag{2.7}$$

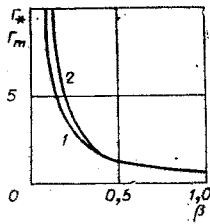


Fig. 3

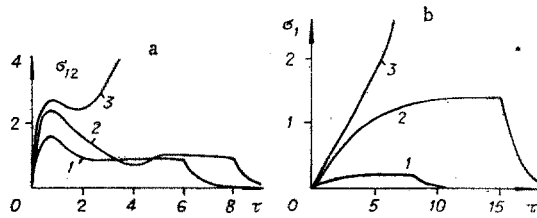


Fig. 4

which corresponds to a purely elastic deformation regime in the present conditions, i.e., to the case of loss of fluidity by the system.

A numerical solution of the system (2.2) with initial conditions (2.3) showed, however, that unlike the above linear-stability conditions, the solution of this problem is unstable and takes on the asymptotic behavior (2.7) for large values of τ if $\Gamma_*(\beta) < \Gamma^0 < \Gamma_m(\beta)$, where $\Gamma_*(\beta)$ is determined from the nonlinear instability of the problem as a whole. Writing $C_{11} \equiv v$, $C_{12} \equiv u$, we can write the system (2.2) in the phase plane u, v :

$$\frac{dv}{du} = v \frac{4\Gamma u - (u^2 + v^2 - 1) \exp\{-\beta v^{-1}[(v-1)^2 + u^2]\}}{2\Gamma(1+u^2) - u(u^2 + v^2 + 1) \exp\{-\beta v^{-1}[(v-1)^2 + u^2]\}} \equiv \frac{\Phi_1(u, v)}{\Phi_2(u, v)} \quad (2.8)$$

The phase portrait of the system (2.2) is shown qualitatively in Fig. 2. The closed curve $\Phi_2 = 0$ corresponds to the vanishing of the denominator, and the curve $\Phi_1 = 0$ corresponds to the vanishing of the numerator on the right side of Eq. (2.8); their points of intersection are stationary points, A and B. We consider the case when the point A is a stable focus and the point B is a saddle. The arrows show the wings of the saddle. In the present case the unstable wing BA of the saddle B is a regular trajectory in a neighborhood of the stationary point A. The stable wings GB and DB of the saddle point B, like the unstable wing BF, go out to infinity. The region of global stability (Σ_-) is situated to the left of the neutral curve GBDE(C), and the region of global instability (Σ_+) is to the right of this curve.

If the point $M(0, 1) \in \Sigma_-$ (which corresponds to the curve of neutral stability GBDE), then the solution of the problem (2.8) is stable. If $M \in \Sigma_+$ (which corresponds to the curve GBDC), then the solution of the Cauchy problem for Eq. (2.8) is unstable irrespective of the stability of the stationary point A. As can be shown by numerical analysis, when the parameter Γ increases, we observe a transition from the situation in which the initial point M is contained in the region of stability to a situation in which it passes into the region of instability; as the parameter Γ increases (for fixed β), the singular points A and B come closer to each other. For every β there exists a critical value $\Gamma_*(\beta)$, when the neutral curve intersects the point M (0, 1). Figure 3 shows $\Gamma_*(\beta)$ in curve 1 and $\Gamma_m(\beta)$ in curve 2. For small values of β the deviation of $\Gamma_*(\beta)$ from $\Gamma_m(\beta)$ may be considerable.

Considerations involving the nonstationary solution of the system (2.2) with the conditions $\Gamma = \text{const}$ ($\Gamma \leq \Gamma_*$) yield the curves of Fig. 4, obtained by numerical integration. In Figs. 4a and 4b we see, respectively, $\sigma_{12}(\tau)$ for $\beta = 0.1$ ($\Gamma_* = 0.981$) and $\sigma_1(\tau)$ for $\beta = 1$ ($\Gamma_* = 0.618$). Curve 1 corresponds to $\Gamma = \Gamma_*/2$; curve 2, to $\Gamma = \Gamma_*$; and curve 3, to $\Gamma = 1.2\Gamma_*$. The left branches of these curves correspond to loading of an elastic liquid from a state of rest with $\Gamma = \text{const}$; the right (descending) branches correspond to relaxation from a stationary regime with $\Gamma \leq \Gamma_*$.

Thus, the question of loss of fluidity of an elastic liquid is related to the existence of a stationary regime of flow from the unperturbed state. For $\Gamma < \Gamma_*(\beta)$ a stationary regime exists. The absence of a stationary regime for $\Gamma > \Gamma_*(\beta)$, with passage to elastic asymptotic behavior (2.7), is treated as loss of fluidity in the system or transition into the highly elastic state. The behavior of the elastic liquid near the transition to the highly elastic state depends substantially on the parameter β , which in [5] was given the physical meaning of the parameter of flexibility of the macromolecules: $\beta \ll 1$ means flexible macromolecules and $\beta \approx 1$ means rigid macromolecules.

For $\beta \approx 1$ in the region of the fluid state there is no accumulation of large reversible deformations in the material. Therefore, the behavior of the elastic liquid, determined from the variation of $\sigma_{12}(\tau)$ when $\Gamma = \text{const} < \Gamma_* \approx \Gamma_m(1)$, as is shown by the numerical solution of the system (2.2), is practically identical with the linear behavior. The normal stresses as $\Gamma \rightarrow \Gamma_*$ increase sharply, but there are no maxima in the prestationary stage of deformation in this case. This is exactly the behavior of polymers with narrow MWD that was observed in [3].

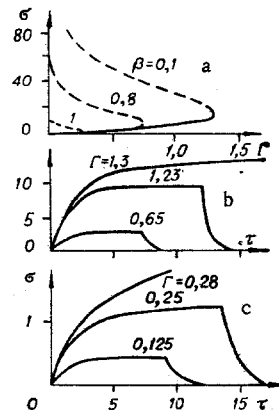


Fig. 5

When $\beta \ll 1$, there may be large reversible deformations in the elastic liquid before the transition to the highly elastic state; as a result of this, for $\Gamma = \text{const} < \Gamma_*$ we observe the entire complex of nonlinear viscoelastic properties described in [7, 8]. For $\Gamma > \Gamma_*(\beta)$ and $\tau \gg 1$ the solution of the system (2.2) is asymptotically described by the formulas (2.7) even for $\beta \ll 1$. It should be noted that while for $\Gamma < \Gamma_*$ ($\beta \ll 1$) the behavior of the system is determined essentially by geometric nonlinearities (as a result of which, for example, the effective viscosity for stationary shear decreases as Γ increases), for $\Gamma \geq \Gamma_*$ there is a sharp increase in the viscosity owing to orientation, notwithstanding the disorienting rotation of particles when shear takes place.

For $\Gamma \geq \Gamma_*$ the orientational phenomena may lead to secondary effects: the formation of crystalline or strongly ordered amorphous regions, depending on the type of polymer. These and other phenomena have been observed experimentally when intensive flow of polymer melts took place in capillaries [9]. When the mechanical effect becomes even more intensive, there may be other phenomena characteristic of solids: cracks in the polymer melt, separation of the polymer from the wall, and intensive slippage near the wall, accompanied by oscillations. Apparently, this set of phenomena has been grouped in the literature under the general name of destruction of a melt [1].

It should also be noted that for $\Gamma > \Gamma_*$ the description of the shear motion of concentrated polymer liquids is made more complicated by the little-investigated problem of the character of the interaction between the polymer and the wall.

§3. Let us consider the case of uniform single-axis tension. The kinematic matrices and the invariants of the tensor \mathbf{C} have the form [$\dot{\gamma} = \dot{\gamma}(t)$ is the longitudinal rate of deformation]

$$\mathbf{e} = \dot{\gamma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{pmatrix}; \quad \boldsymbol{\omega} = 0;$$

$$\mathbf{C} = \begin{pmatrix} \lambda^2 & 0 & 0 \\ 0 & \lambda^{-1} & 0 \\ 0 & 0 & \lambda^{-1} \end{pmatrix}; \quad \mathbf{C}^{-1} = \begin{pmatrix} \lambda^{-2} & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}; \quad (3.1)$$

$$I_1 = \text{sp}\mathbf{C} = \lambda^2 + 2\lambda^{-1}; \quad I_2 = \text{sp}\mathbf{C}^{-1} = 2\lambda + \lambda^{-2} \quad (\lambda > 1).$$

Making use of (1.8), (1.9), and (3.1), we obtain from the kinematic formulas (1.2) and (1.3) a scalar equation for the variation of $\lambda(\tau)$:

$$\frac{6}{\lambda} \frac{d\lambda}{d\tau} + \frac{(\lambda^2 - 1)(\lambda^2 + \lambda + 1)}{\lambda^2} \exp\left\{-\frac{\beta}{2\lambda^2} (\lambda - 1)^2 (\lambda^2 + 4\lambda + 1)\right\} = 6\Gamma(\tau) \quad (\tau = t/\theta_0, \quad \Gamma = \dot{\gamma}\theta_0). \quad (3.2)$$

Making use of the dynamic condition on the lateral surface of a cylindrical rod (we are considering the inertia-free formulation of the problem), we obtain an expression for the total dimensionless axial stress in a cross section of the specimen

$$\sigma = (\sigma_{11} - p)/2\mu = \lambda^2 - \lambda^{-1}. \quad (3.3)$$

Formula (3.3) agrees with the analogous formula obtained in the linear theory of rubber elasticity [6].

For Eq. (3.2) we consider two deformation regimes analogous to simple shear: transition to stationary longitudinal flow with $\Gamma = \text{const}$ from a state of rest with $\lambda(0) = 1$ and relaxation with $\lambda(\tau_0) = \lambda_0$.

A stationary solution of (3.2) exists only for $\Gamma < \Gamma_*^0(\beta)$ and has the same properties as are found in simple shear. Here $\Gamma_*^0(\beta)$ is the critical value of the longitudinal rate of deformation such that for $\Gamma < \Gamma_*^0(\beta)$ there is stationary flow, while for $\Gamma > \Gamma_*^0(\beta)$ there is no stationary flow. It follows from (3.2) that the variation of $\Gamma_*^0(\beta)$ is determined as $\max_{\lambda} \Gamma^0(\lambda, \beta)$ where $\Gamma^0(\lambda, \beta) = [(\lambda^2 - 1)(\lambda^2 + \lambda + 1)/6\lambda^2] \exp \{ -(\beta/\lambda^2)(\lambda - 1)^2(\lambda^2 + 4\lambda + 1) \}$.

The stationary curves for $\sigma(\Gamma)$ are shown in Fig. 5a; the upper branches of these curves, shown in dashed form, are unstable. The critical values of the longitudinal rate of deformation Γ_*^0 are less than the critical values of the rate of deformation for simple shear for the same material; this means that for simple tension we have the most optimal conditions of orientation, not accompanied (as in simple shear) by the disorienting influence of the rotation of the particles of the material.

For $\Gamma > \Gamma_*^0$ ($\Gamma = \text{const}$) and $\tau \gg 1$ we obtain from (3.2) the asymptotic formulas

$$\lambda \approx \exp(\Gamma\tau); \quad \sigma \approx \exp(2\Gamma\tau),$$

which correspond to a pure elastic regime of deformation after loss of fluidity in the system.

The nonstationary curves for $\sigma(\tau)$, corresponding to transition to steady-state flow and relaxation with steady-state flow ($\Gamma < \Gamma_*^0(\beta)$), and the curves of $\sigma(\tau)$ for $\Gamma > \Gamma_*^0(\beta)$ are shown in Figs. 5b and 5c (5b corresponds to $\beta = 0.1$; 5c to $\beta = 1$). It is interesting to note that, just as for the case of simple shear, for $\beta = 1$ we observe a practically linear behavior of the elastic liquid up to $\Gamma = \Gamma_*^0$, which was observed in experimental studies on single-axis tension [2] carried out on polymers with narrow MWD. For $\Gamma > \Gamma_*^0(\beta)$ the rapid increase of stresses with respect to time must lead to failure of the specimen by a mechanism characteristic of vulcanized rubbers.

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